Linking Impedance Simulations to Beam Dynamics Simulations

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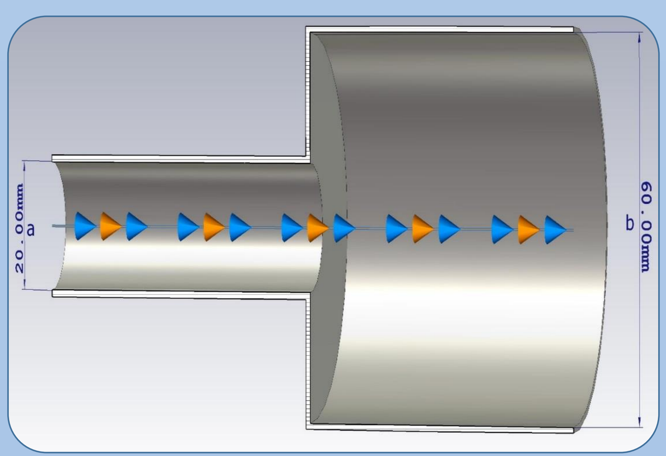
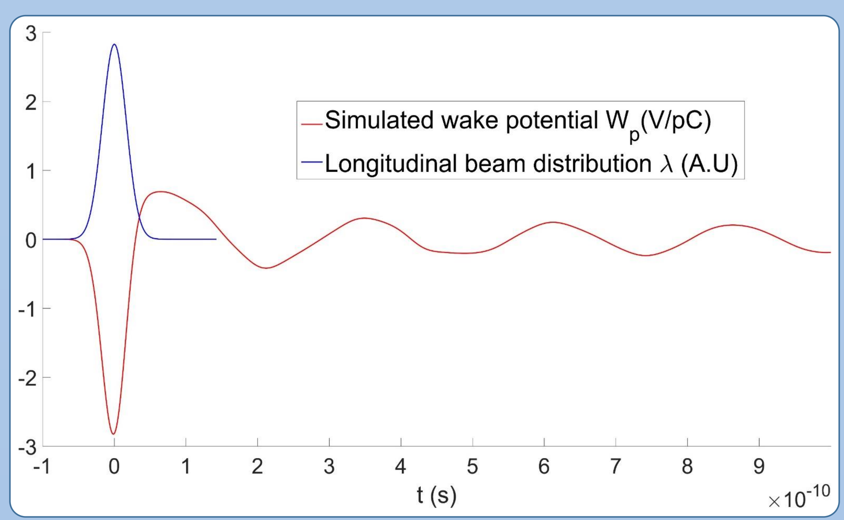
Abstract: An important source of instability in accelerators is the beam coupling impedance. Impedance simulations can be useful to feed beam dynamics studies aiming to understand the encountered instabilities and limitations due to impedance. But the beam dynamics simulations results depends on the method used to link impedance simulations with beam dynamics. A method based on wakefield theory and applied as post processing on wake potential simulations is proposed and compared with several known methods. This method has been tested for several accelerator components, both with short range wakefield and long range wakefield, and allowed us to compute the wake potential for any beam longitudinal distribution.

Wakefield reconstruction for a analytical case: the step out

- Using solvers like CST Particle Studio it is possible to simulate the wake potential W_p for a given longitudinal beam distribution λ. An example in the case of a step out transition:

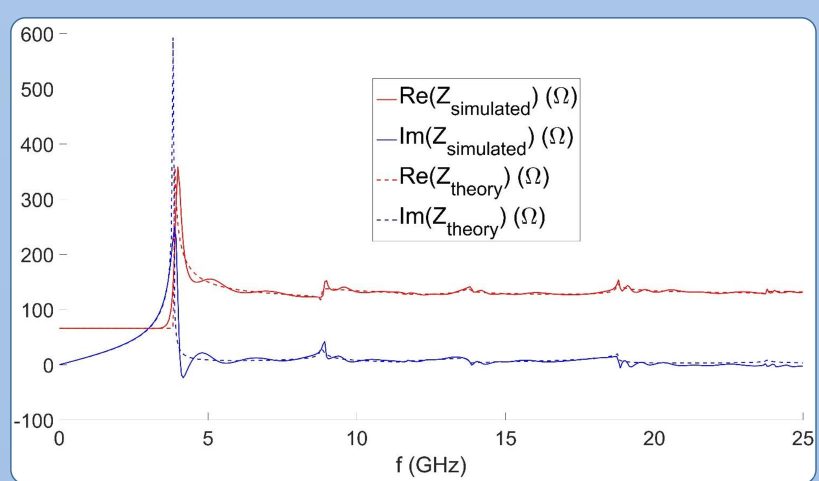




- For a step out transition the analytical impedance is known[1] and can be compared to the impedance obtained by wakefield simulations.

$$Z_{theory}(k) = -\frac{Z_0 a}{\pi b}\sum_{n=1}^{+\infty}\left[g_n^+(kb-\lambda_{bn}) - \frac{b}{a}g_n^-(ka+\lambda_{an})\right]$$

With: $k = \frac{2\pi f}{c}$, $Z_0 \approx 376\,\Omega$
a, b the pipes dimensions
$\lambda_{an}, \lambda_{bn}$ coefficients function of a, b and n
g_n^+, g_n^- coefficients function of a, b, n and k

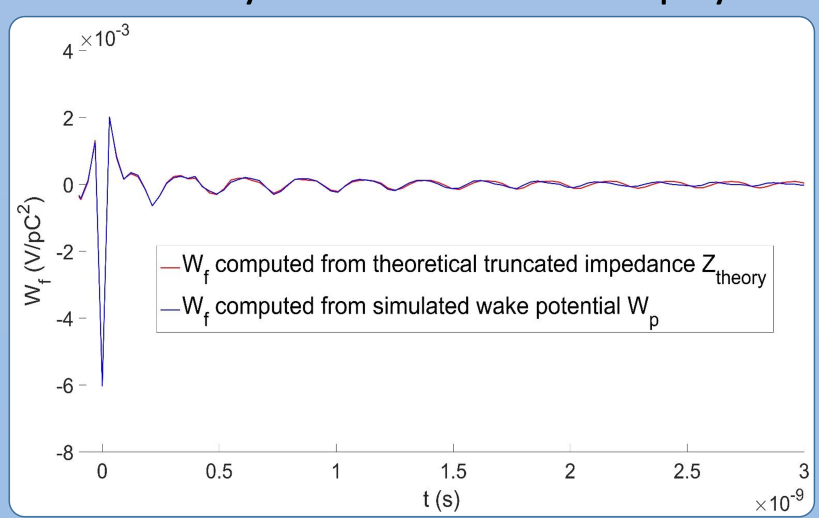


- A simple application of wakefield theory gives a way to extract the wake function from the simulated wake potential. In practice the application of this formula can be tricky and numerical tips may be needed to obtain satisfying results in most case. The wake function is computed up to a maximum frequency f_{max} and is not valid beyond without more physical approximation.

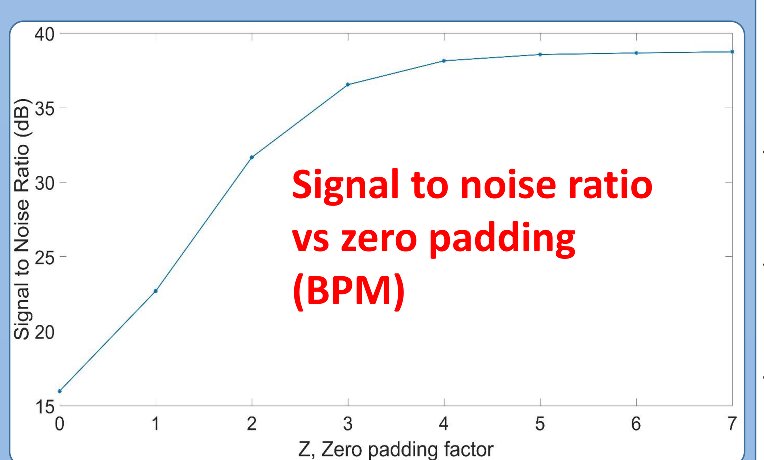
$$W_p(t) = \int_{-\infty}^{+\infty} W_f(t-x)\lambda(x)dx$$

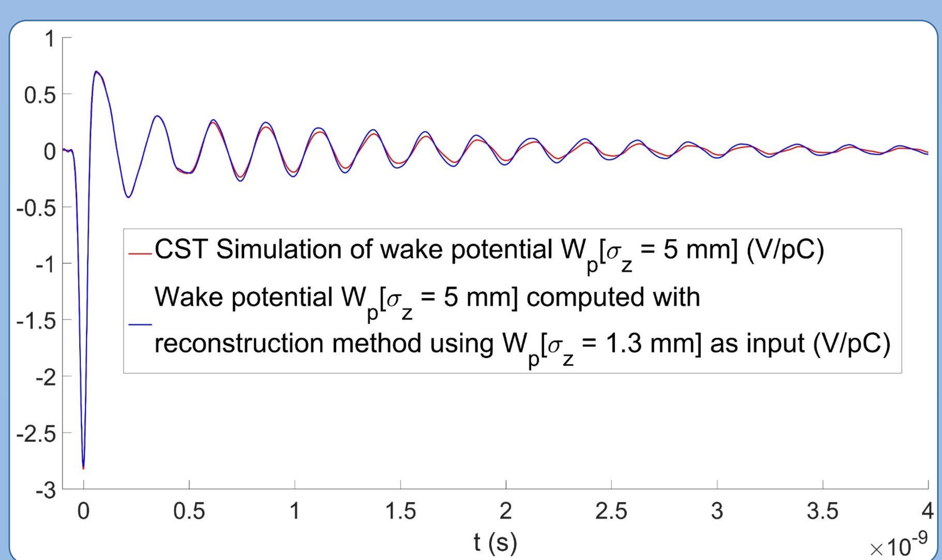
$$W_f(t) = FT^{-1}\left[\frac{FT[W_p(t)]}{FT[\lambda(t)]}\right]$$

$$\lim_{f_{max}\to+\infty} W_f(t) \approx -\frac{Z_0}{Q\pi}\ln\frac{b}{a}\,\delta(t)$$



- The wake function can now be used to compute the wake potential for any longitudinal beam distribution λ.





- The wake function can be improved in some case by using physical approximation like considering that the impedance vanish for very high frequencies, which is true for most realistic elements. In practice this amounts to use zero padding before computing the FFT.

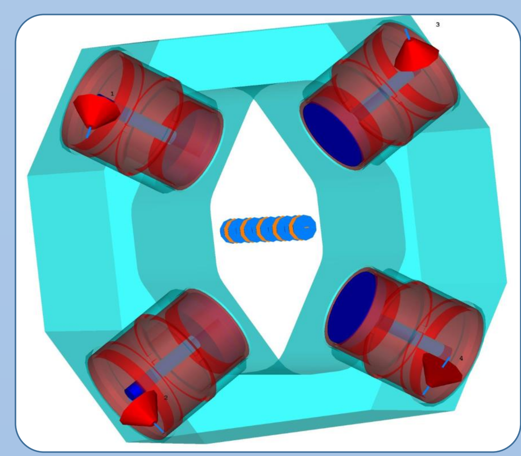
Comparaison for a real case: ThomX BPM

Most of the beam dynamics simulations that include wakefield effects use a wake function which is convoluted with the beam longitudinal distribution at each step to compute the resulting wake potential. The way to compute the wake function is usually what differ between different code:

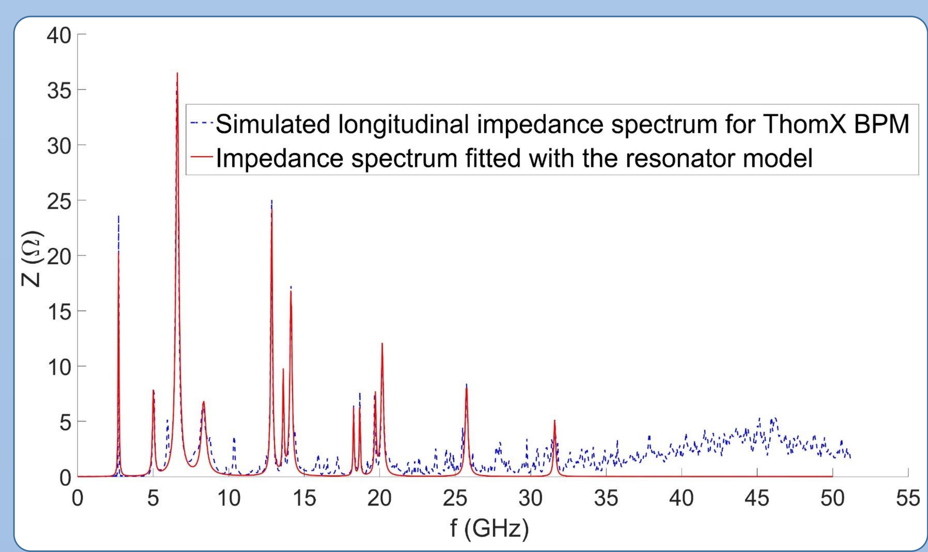
- The resonator wake model[2] can be used to fit an impedance spectrum with a sum of parallel RLC resonators and compute the resulting wake function.

$$Z(\omega) = \sum_i \frac{R_i}{1 + jQ_i\left(\frac{\omega}{\omega_i} - \frac{\omega_i}{\omega}\right)}$$

$$W_f(t) = \sum_i \frac{\omega_i R_i}{Q_i}\exp\left(-\frac{\omega_i t}{2Q_i}\right)\left[\cos\omega_i' t - \frac{1}{2Q_i'}\sin\omega_i' t\right]$$



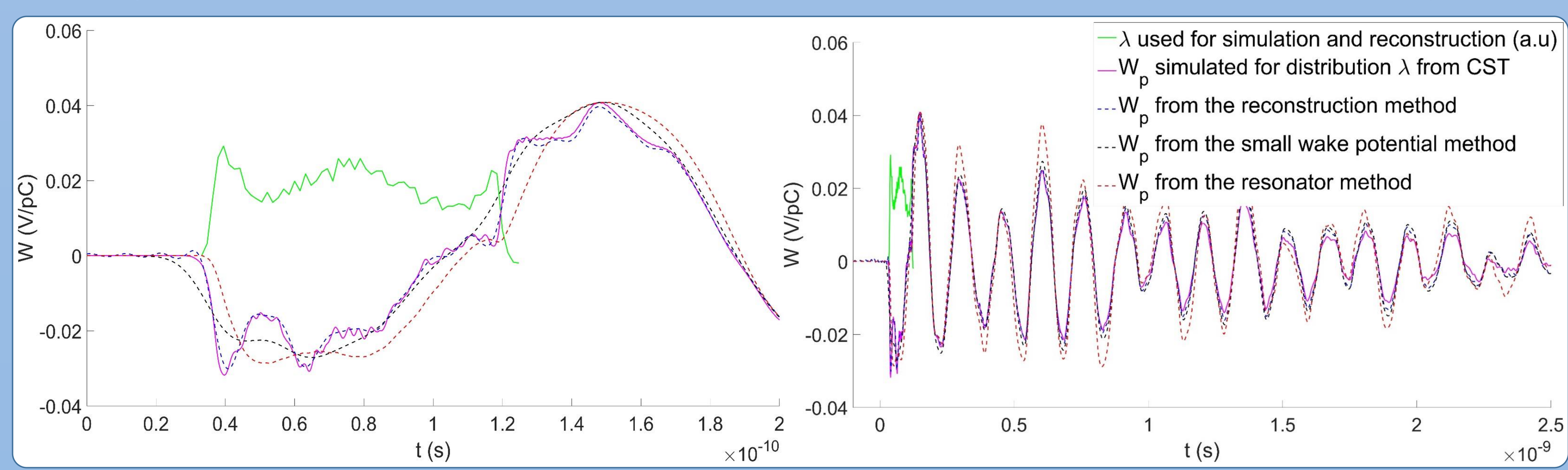
ThomX Beam Position Monitor (BPM)



With:
R_i the shunt impedance
Q_i the quality factor
ω_i the resonant frequency
$Q_i' = \sqrt{Q_i - 1/4}$,
$\omega_i' = \omega_i Q_i'/Q_i$

- One other possibility is to simulate the wake potential with a small longitudinal distribution and use this wake potential as a wake function[3]. Ideally this method should be used with a simulated bunch length σ at least ten times smaller than the wake potential bunch length σ_z.

A comparison of the three methods is shown for the case of ThomX BPM, a wake potential simulation for a Gaussian distribution of $\sigma_z = 2mm$ is used as the same input for the three methods. The wake potential W_p computed by a CST simulation (in pink) for the test distribution λ (in green) is compared to wake potentials obtained for λ with the different methods (in dotted lines).



The likeness of the wake potential reconstruction is evaluated by computing the Signal to Noise Ratio (SNR) over 1 ns. The wake potential simulated by CST is considered as the signal, the difference between the CST wake and the reconstructed wake is considered as the noise.

Conclusion

This method shows an improved ability to produce EM simulation-like wakes for particle dynamics simulations. It can, for example, be used to see fine impedance effects induced by complex bunch shape. This method has also the advantage of being usable for any type of wake.

Method	Reconstruction postprocessing	Small wake potential	Resonator wake
SNR (dB) for this case	26	16	8
Work for high frequecies	Yes	Difficult, need to simulate very high frequencies	Yes
Long-range wakefield	Yes	Yes	Yes
Short-range wakefield	Yes	Yes	No, impossible to fit the impedance

References

- [1] – S. A. Kheifets, S. A. Heifets, Radiation of a charge in a perfectly conducting cylindrical pipe with a jump in its cross section, SLAC-PUB-3965, May 1986.
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